**Landau Zener Problem in Quantum Annealing**

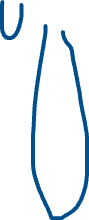
* Max weighted independent set problem
* Define system – Hamiltonian, interacting spins, computational, problem
* Introduce catalyst, discuss results
* Landau Zener transition potentially?

The max weighted independent set problem is an optimisation problem that may be solved with simulated and quantum annealing (QA) methods. Given a network, this problem aims to find the largest set of vertices that are not connected to each other. We chose to study the cases of complete bipartite networks, which have 2 disjoint and independent sets U and V, with n and n-1 vertices in each edge respectively, where any vertex in U is connected to all vertices in V, and vice versa. Since there are 2 trivial solutions to the problem they are a good candidate for understanding QA and finding the true minimum of the problem. The n = 3 example of a complete bipartite graph is the following:



A picture containing line, diagram, design

Description automatically generated



<https://en.wikipedia.org/wiki/Bipartite_graph>

<https://mathworld.wolfram.com/CompleteBipartiteGraph.html>

We may use quantum mechanics to solve this problem by modelling the vertices as interacting spin-1/2 particles, with the Hamiltonian

Where the first sum is over vertices and the second is over edges is this the Pauli z matrix associated with the ith vertex, is the associated weight, is the coupling strength (identical between all spins) and is shorthand for , i.e. the coupling of vertices i and j . The details of the mapping from the solution to the Hamiltonian are not important, but is done such that any subset including a coupling invokes a severe penalty, thus is not the lowest energy solution. Each eigenstate of the is a possible solution, and thus we are interested in finding the energy

To find the lowest energy solution, quantum annealing is implemented by preparing a system in the known ground state of the Hamiltonian

Where is the Pauli x matrix associated with the ith vertex.

The anneal process begins by evolving from an initial ends in , such that

where increases linearly from 0 to 1.

The spectrum of includes a perturbative crossing between the ground state and first excited state, which results in the anneal always being in the first excited state. Although the adiabatic theorem states that a slow enough anneal would stay in the ground state, this is not computationally feasible.

This may be mitigated by introducing a ‘catalyst’ Hamiltonian that vanishes at the start and end of the anneal, . The Hamiltonian now becomes

.

The Catalyst is chosen to be an xx-coupling

Where is the coupling strength between 2 spins, the ‘catalyst strength’. For the problem studied, the spins are indistinguishable, and thus enumeration is included for completeness. A sketch of the interacting spins is the following (values of aren’t important):

A picture containing diagram, line, circle

Description automatically generated

The consequence of introducing the catalyst is seen in the spectrum of , which forms a closing gap (NOT the same as a perturbative crossing). In the diagram below, the energy difference between the ground state and first excited state of the n=3 system is shown with different catalyst strengths (values of , demonstrating its formation:



A picture containing line, text, plot, diagram

Description automatically generated



The probability of the state being in the ground state and first excited state at the corresponding catalyst strength in the previous figure is shown below, demonstrating the jump of the state into the first excited state for a chosen anneal time of 200ns:

A picture containing text, diagram, line, parallel

Description automatically generated

We now change our attention to the n=5 case (9 spins overall, what Natasha has studied in the interim), and consider the ground state probability as a function of total anneal time T, at different choices of . For non-optimal values, this results in an increase followed by an exponential decay, as demonstrated in the following example:

A graph with different colored lines

Description automatically generated with low confidence

Here is the optimum catalyst strength (minimum closing gap), and is normalised difference between and , as defined in the diagram. (Fidelity = ground state probability). The exponential decay is of interest here, and we would like to know how this is parameterised, with Landau Zener transition exponentials being a potential candidate.

At the point in the spectrum of the closing gap, there is a probability of the quantum state tunnelling into the first excited stated, given by the probability is defined below:

A picture containing text, line, diagram, screenshot

Description automatically generated

The landau Zener formula tells us that

Where , the minimum energy difference between the ground and first excited state (not sure if this at the closing gap or perturbative crossing, need to ask Natasha).

Using the chain rule, can be expressed as a function of total anneal time

Where , the normalised anneal time.

It is hypothesised that the exponential decay of the ground state probability as a function of at the end of the anneal is related to the landau Zener exponent, but the question lies in finding the constant of proportionality for , bespoke to the system Hamiltonian and closing gap.

**i.e. the problem is to find what the value of is for our system.**